Signal and Image Processing - Assignment 1

Assigned March 5th **Due March 17th 23:59**

The point of this homework is to see the kind of mathematics that are assumed by the lectures and course materials.

You are to complete this individually. For this assignment you do not need, and must not use, a calculator or a computer program (Matlab, Python, etc.) to compute the answers; only use a computer to write it up and upload it to Moodle. The work of answering these questions should be done by you, **in your head or "by hand"**. The questions test mathematical knowledge and insight of the sort assumed in this class, so you are doing yourself a disservice if you use a calculator or computing program.

If doing this assignment takes more than about two hours, or if you need to consult many web pages or textbooks to learn the material to find the answers, your math background may not be a good fit for this class, and this class may not be a good fit for you.

There are enough questions, and the questions are simple enough, that we will not be giving partial credit for incorrect answers. For this homework, we are looking for short answers, not explanations (future assignments may require more explanatory writing). A maximum of **25 points** is awarded for this assignment — **all questions have equal weight**.

You should upload a PDF document with your solutions, whether this is hand written and scanned, or written on a computer does not matter, as long as it is legible.

Time you spent on this assignment: ____ hours

1. Let A, B and C be invertible matrices. What is $(ABC)^T$, in terms of A^T, B^T and C^T?

2. (same A, B, C as previous) What is $(ABC)^{-1}$, in terms of A^{-1}, B^{-1} and C^{-1}?

3. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$. What is A^TA and AA^T? (write out the 2×2 matrix of integers)

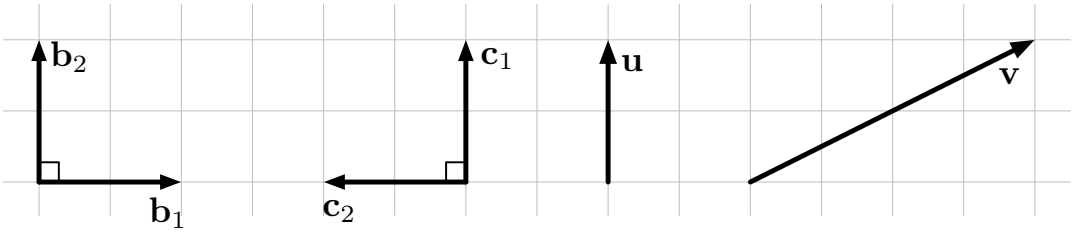
4. Are A^TA and AA^T both always symmetric, for any square of A? (yes or no)

5. Let $u = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. What is u^Tv and uv^T?

6. Let $u = \begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix}$, $A = \begin{bmatrix} 3 & -10 & 6 \\ 2 & -1 & 4 \\ -2 & -16 & -4 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 2 \\ 4711 \end{bmatrix}$. What is $u^T Av$?

7. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -2 & -1 \\ 1 & 1 & -2 \\ 0 & 0 & 5 \end{bmatrix}$ and $v = \begin{bmatrix} \pi \\ \sqrt{-1} \\ 42 \end{bmatrix}$. What is ABv?

8. The following figure shows two bases $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ and two vectors $\mathbf{u}$ and $\mathbf{v}$ drawn to scale on a regular grid.



   The matrix representation of $\mathbf{x}$ in basis $\mathcal{B}$, notated $[\mathbf{x}]_\mathcal{B}$, is the column vector of coordinates $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ for which $\mathbf{x} = x_1\mathbf{b}_1 + x_2\mathbf{b}_2$ (as example: $[\mathbf{u}]_\mathcal{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$).

   What is $[\mathbf{u} + \mathbf{v}]_\mathcal{B}$?

9. (same set-up as above) What is $[\mathbf{u} - \mathbf{v}]_\mathcal{B}$?

10. (same set-up as above) What is $[\mathbf{u} + \mathbf{v}]_\mathcal{C}$?

11. A function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^n$ for some integer n. For what n is f continuous?

12. (same f as previous) For what n is f monotonic?

13. A function $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} ax + b & x < 0 \\ (x-1)^2 & x \geq 0 \end{cases}$$

   for some $a, b \in \mathbb{R}$. Give an example of specific numeric values for a and b that make f discontinuous. (answer in the form "$a =$ $\quad, b =$ $\quad$")

14. (same f as previous) Give values for a and b so that f is C^0 but not C^1 continuous (these are different orders of continuity).

15. (same f as previous) Give values for a and b so that f is C^1 continuous.

16. To a first-order approximation (in the sense of a Taylor series expansion around 0) what is $\sin 0.01$?

17. Let $f(x) = x^3 + \cos x$. To a first-order approximation (again in the Taylor series sense), what is $f''(\epsilon)$ for ϵ near 0?

18. Let $f(x) = 1 - |x|$. What is $\int_{-1}^{1} f(x)dx$?

19. Let $f(x) = \sqrt{1 - x^2}$. What is $\int_{-1}^{1} f(x)dx$? (hint: this is more about geometry than calculus)

20. What is

$$\lim_{n\to\infty} \sum_{m=0}^{n-1} \frac{m}{n^2}$$

where n is an integer?

21. Express $z = 2 - i$ in exponential and polar form.

22. Express $z = 6$ in exponential and polar form.

23. Express $z = 3e^{i\pi/6}$ in Cartesian and polar form.

24. Express $z = e^{i\pi/4}$ in Cartesian and polar form.

25. Express $z = cos(\pi) + i\ sin(\pi)$ in Cartesian and exponential form.

Most of the problems in this assignment are directly taken or modified versions of problems from the Scientific Visualization course at the University of Chicago by Gordon Kindlmann.